Lecture 20
Reinforcement Learning
(Part II)

Lei Li, **Yu-Xiang Wang**

Announcements

1. **Poster printing:** See the poster printing instructions from edstem.

2. **Project Presentation day:** Small prizes will be given to the best poster presentations. You can invite your friends to come！

3. **Course evaluation:** Please complete you ESCI surveys if you haven’t yet. It takes only a few minutes.

Optional HW4

- For practice problems in convex optimization
  - For a simple coding problem with cvx: Q4 of here with data here
  - For practices on convex analysis: Q1,2,3 here

- Theory / concept practices for RL:
  - Problem 3 and 4 here
  - Problem 1 and 2 here
- Coding practice for MDP / RL: Here

- For more advanced problems in RL:
  - see HW1,2,3 from my RL theory course.
  - These are only useful if you are hoping to do RL research.

Recap: Markov Decision processes (infinite horizon / discounted)

- Infinite horizon / discounted setting

$$\mathcal{M}(\mathcal{S}, \mathcal{A}, P, r, \gamma, \mu)$$

Transition kernel: $P: S \times A \to \Delta(S)$ i.e. $P(s'|s,a)$

(Expected) reward function: $r: S \times A \to \mathbb{R} / [0, R_{max}]$ $\mathbb{E}[R_t \mid S_t = s, A_t = a] =: r(s,a)$

Initial state distribution $\mu \in \Delta(S)$

Discounting factor: γ

Stationary Policy π: mapping from state to an action (possibly a random action).

Recap: Value functions

- state value function: $V^{\pi}(s)$
  - expected **long-term** return when starting in *s* and following π

$$V^{\pi}(s) = \mathbb{E}_{\pi}[R_1 + \gamma R_2 + ... + \gamma^{t-1} R_t + ...|S_1 = s]$$

- state-action value function: $Q^{\pi}(s,a)$
  - expected **long-term** return when starting in *s*, performing *a,* and following π

$$Q^{\pi}(s,a) = \mathbb{E}_{\pi}[R_1 + \gamma R_2 + ... + \gamma^{t-1} R_t + ...|S_1 = s, A_1 = a]$$

Recap: Bellman equations

- Bellman consistency equation

$$V^\pi(s) = \sum_a \pi(a|s) \sum_{s'} P(s'|s,a)[r(s,a,s') + \gamma V^\pi(s')]$$

$$\underline{V^\pi = r^\pi + \gamma P^\pi V^\pi}$$

- Bellman optimality equation

$$V^*(s) = \underbrace{\max_a}_{} \sum_{s'} P(s'|s,a)[r(s,a,s') + \gamma \underbrace{V^*(s')}_{})]$$

$$\boxed{Q^* = r + \gamma P V_{\underset{\uparrow}{Q^*}}}$$ where $$V_Q(s) := \underbrace{\max_{a \in \mathcal{A}} Q(s,a)}_{}.$$

Recap: MDP planning and Value iterations

- MDP planning:

$$\text{Find } \pi^* \text{ such that } V^{\pi}(s) = V^*(s) \quad \forall s$$

$$\pi \text{ is } \epsilon\text{-optimal if } \underline{V^{\pi} \geq V^*(s) - \epsilon \mathbf{1}}$$

- Policy evaluation
  - Solving Bellman consistency equation $\Rightarrow V^{\pi}$

$$\underline{\pi_{next} = \arg\max Q^{\pi}}$$

- Value iteration
  - Solving Bellman optimality equation

Recap: RL agent needs to learn the underlying MDP model

- **Model-based algorithm**
  - Estimates the MDP then do MDP planning

- **Model-free algorithms** Q^* only thing needed
  - Monte Carlo Policy evaluation + Policy improvement
  - Temporal difference learning = MC + Bellman equations

Recap: TD Learning

- TD-Policy evaluation

$$V(S_t) \leftarrow V(S_t) + \alpha \Big[\underbrace{\overset{\checkmark}{R_{t+1}} + \gamma \overset{\text{if } V = V^{\pi}}{\boxed{V(S_{t+1})}} - V(S_t)} \Big]$$

- TD-Policy optimization
  - SARSA (on-policy)

$$Q(S, A) \leftarrow Q(S, A) + \alpha \underbrace{\big[R + \gamma \overset{\checkmark}{Q}(S', A') - Q(S, A) \big]}$$

Then choose the next A’ using Q, e.g., eps-greedy.

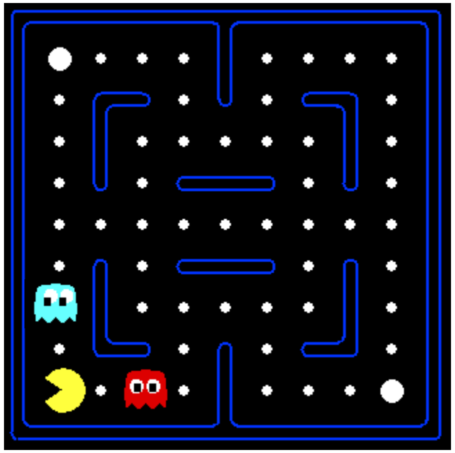
  - Q-Learning (off-policy)

$$Q(S, A) \leftarrow Q(S, A) + \alpha \big[R + \gamma \underbrace{\max_a Q(S', a)} - Q(S, A) \big]$$

Then choose the next action in your favorite way.

Recap: The problem of large-state space

Let's say we discover through experience that this state is bad:



(From Dan Klein and Pieter Abbeel)

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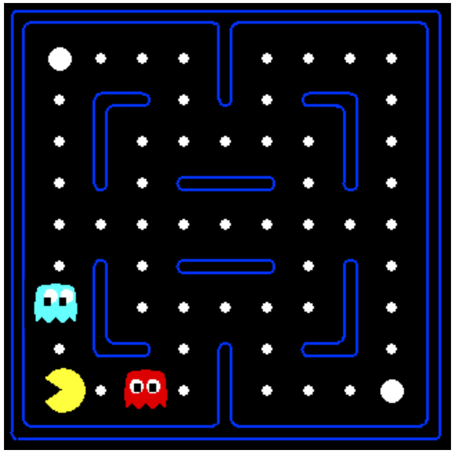
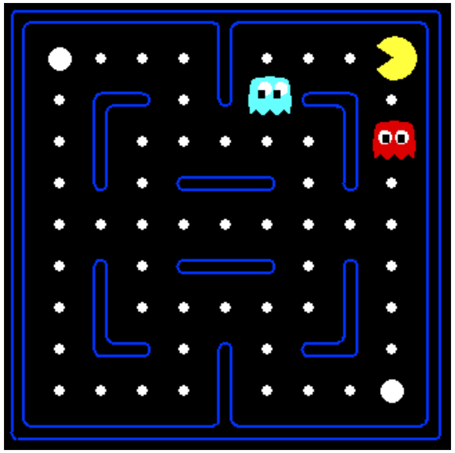


In naïve q-learning, we know nothing about this state:



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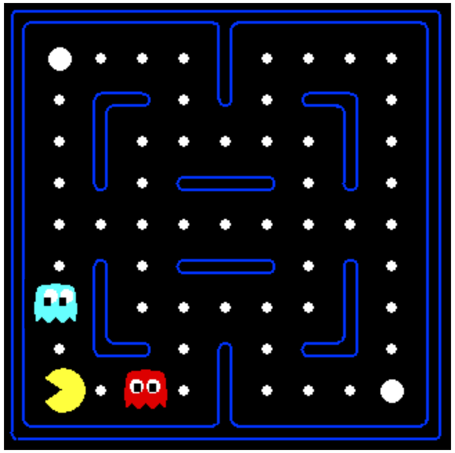

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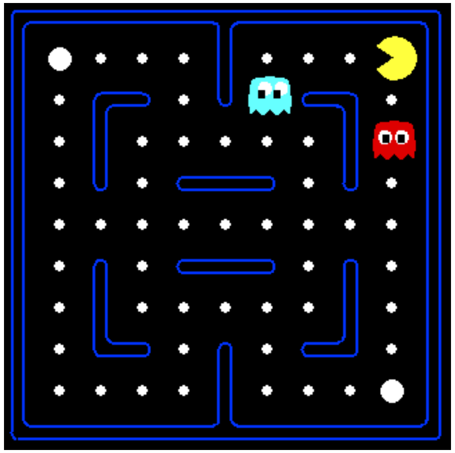
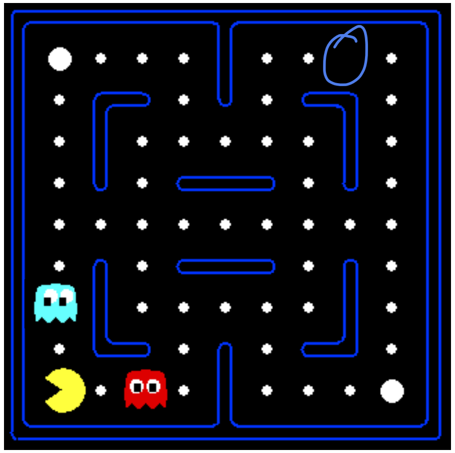


Or even this one!



(From Dan Klein and Pieter Abbeel)

This lecture

- Solve the problem of large state space with function approximation

- Other RL algorithms: Policy gradient

- Exploration in RL

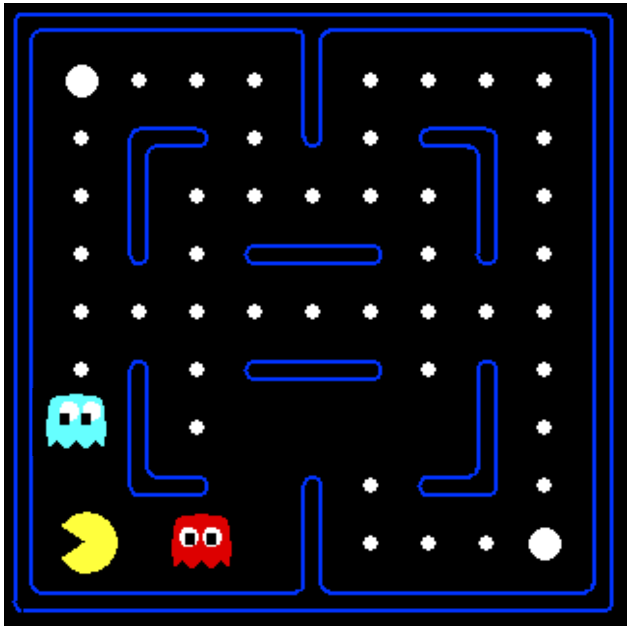
Video of Demo Q-Learning Pacman – Tiny – Watch All

Video of Demo Q-Learning Pacman – Tiny – Silent Train

Video of Demo Q-Learning Pacman – Tricky – Watch All

Why not use an evaluation function? A Feature-Based Representations

- Solution: describe a state using a vector of features (properties)
  - Features are functions from states to real numbers (often 0/1) that capture important properties of the state
  - Example features:
    - Distance to closest ghost
    - Distance to closest dot
    - Number of ghosts
    - 1 / (dist to dot)^2
    - Is Pacman in a tunnel? (0/1)
    - …… etc.
    - Is it the exact state on this slide?
  - Can also describe a q-state (s, a) with features (e.g. action moves closer to food)



Linear Value Functions

- Using a feature representation, we can write a q function (or value function) for any state using a few weights:
  - $V_{\mathbf{w}}(s) = w_1 f_1(s) + w_2 f_2(s) + \ldots + w_n f_n(s)$ $= \langle w, \vec{f}(s) \rangle$
  - $Q_{\mathbf{w}}(s,a) = w_1 f_1(s,a) + w_2 f_2(s,a) + \ldots + w_n f_n(s,a)$ $= \langle w, \vec{g}(s,a) \rangle$

$g(s,a) = E_{s' \sim P(s,a)} f(s')$

- Advantage: our experience is summed up in a few powerful numbers

- Disadvantage: states may share features but actually be very different in value!

Updating a linear value function

Updating a linear value function

- Original Q learning rule tries to reduce prediction error at s, a:

$$Q(s,a) \leftarrow Q(s,a) + \alpha \cdot [R(s,a,s') + \gamma \max_{a'} \underline{Q(s',a')} - Q(s,a)]$$

Updating a linear value function

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$$Q(s,a) \text{ ⍰ } Q(s,a) + \alpha \cdot [R(s,a,s') + \gamma \max_{a'} Q(s',a') - Q(s,a)]$$

- Instead, we update the weights to try to reduce the error at s, a:

Updating a linear value function

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$$Q(s,a) \leftarrow Q(s,a) + \alpha \cdot [R(s,a,s') + \gamma \max_{a'} Q(s',a') - Q(s,a)]$$

$$\langle w, f(s,a) \rangle \leftarrow \langle w, f(s,a) \rangle + \alpha \left[R + \gamma \max_{a'} \langle w, f(s',a') \rangle - \langle w, f(s,a) \rangle \right)$$

- Instead, we update the weights to try to reduce the error at s, a:

$$w_i \leftarrow w_i + \alpha \cdot [R(s,a,s') + \gamma \max_{a'} Q(s',a') - Q(s,a)] \, \partial Q_{\mathbf{w}}(s,a)/\partial w_i$$

$$= w_i + \alpha \cdot \underbrace{[R(s,a,s') + \gamma \max_{a'} Q(s',a') - Q(s,a)]}_{\uparrow} f_i(s,a)$$

Updating a linear value function

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$$= w_i + \alpha \cdot \underbrace{[R(s,a,s') + \gamma \max_{a'} Q(s',a')}_{} - \underset{\uparrow}{\underline{Q(s,a)}}]\, f_i(s,a)$$

- Qualitative justification:
  - Pleasant surprise: increase weights on positive features, decrease on negative ones
  - Unpleasant surprise: decrease weights on positive features, increase on negative ones

PACMAN Q-Learning (Linear function approx.)

Deriving the TD via incremental optimization that minimizes Bellman errors

$\hat{V}^{\pi}(s,w)$ w: weights

- Mean Square Error and Mean Square Bellman error

$$\min_w \mathbb{E}_{s\sim\mu}\left[\left(V^{\pi}(s) - \hat{V}^{\pi}(s;w)\right)^2\right]$$

Monte Carlo G_i s.t. $\mathbb{E}[G_i] = V^{\pi}(s)$

$$\min_w \sum_{i=1}^{N}\left(G^{s_i} - \hat{V}^{\pi}(s_i,w)\right)^2$$

$$w^{+} = w - \alpha \nabla_w \left(G_i^{s_i} - \hat{V}^{\pi}(s_i,w)\right)^2$$

$$= w - \alpha\left(\hat{V}^{\pi}(s_i,w) - G_i^{s_i}\right)\cdot \nabla_w \hat{V}^{\pi}(s_i,w)$$

$$= w + \alpha\left(G_i^{s_i} - \hat{V}^{\pi}(s_i,w)\right)\nabla_w \hat{V}^{\pi}(s_i,w)$$

SGD

$$\min_w \mathbb{E}_{s\sim\mu}\left[\left(r^{\pi}(s) + \gamma\, \mathbb{E}_{\substack{s'\sim P(s,a)\\ a\sim\pi(s)}} \hat{V}^{\pi}(s';w) - \hat{V}^{\pi}(s,w)\right)^2\right]$$

Sample $s_i \sim \mu$, $s_i' \sim P(s_i, a_i)$
R_i $a_i \sim \pi(s_i)$.

$$\min \frac{1}{n}\sum_{i=1}^{n}\left(R_i + \gamma \underbrace{\hat{V}^{\pi}(s_i';w)}_{\text{fix,}} - \underbrace{\hat{V}(s_i,w)}_{\nabla \text{ only}}\right)^2$$

$$w^{+} = w - \alpha \nabla_w \left(\underbrace{R_i + \gamma V}\, - \hat{V}(s_i,w)\right)^2$$

$$= w + \alpha\left(R_i + \gamma V - \hat{V}(s_i,w)\right)\cdot \nabla \hat{V}(s_i,w)$$

"Semigradient update"

So far, in RL algorithms

- Model-based approaches
  - Estimate the MDP parameters.
  - Then use policy-iterations, value iterations.

- Monte Carlo methods:
  - estimating the rewards by empirical averages

- Temporal Difference methods:
  - Combine Monte Carlo methods with Dynamic Programming

- Linear function approximation in Q-learning
  - Similar to SGD
  - Learning heuristic function

Policy class and policy gradient methods

$$\boxed{\pi_w(s,a) = \frac{e^{f(s,a)^T w}}{\sum_{a'} e^{f(s,a')^T w}}}$$

- Policy $\pi \in \Pi$
- Parametric policy class:

$$\underline{\Pi = \{\pi_\theta | \theta \in \mathbb{R}^d\}}$$

- Goal: optimize the value

$$V^{\pi} = V^{\pi_\theta} \qquad \underline{\theta^* = \arg\max_{\theta \in \mathbb{R}^d} V^{\pi_\theta}}$$

- Policy gradient methods
  - aim at learning the policy parameter by SGD.

Policy gradient

- Objective function to maximize: $J(\boldsymbol{\theta}) \doteq v_{\pi_{\boldsymbol{\theta}}}(s_0),$

*Note how this theorem is non-trivial… The first two terms depends on π, but we did not take the gradient w.r.t. them.

Policy gradient

- Objective function to maximize: $J(\boldsymbol{\theta}) \doteq v_{\pi_{\boldsymbol{\theta}}}(s_0),$
- Do SGD: $\boldsymbol{\theta}_{t+1} = \boldsymbol{\theta}_t + \alpha \widehat{\nabla J(\boldsymbol{\theta}_t)},$

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- Policy gradient theorem:

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Policy gradient

- Objective function to maximize: $J(\boldsymbol{\theta}) \doteq v_{\pi_{\boldsymbol{\theta}}}(s_0)$,
- Do SGD: $\boldsymbol{\theta}_{t+1} = \boldsymbol{\theta}_t + \alpha \widehat{\nabla J(\boldsymbol{\theta}_t)}$,
- Policy gradient theorem:

$(ab)' = a'b + ab'$

$J(\theta) = \sum_s d^\pi(s) \sum_a \pi(a|s;\theta) \, Q^\pi(s,a)$

$$\nabla J(\boldsymbol{\theta}) = \sum_s d^\pi(s) \sum_a Q^\pi(s,a) \nabla_{\boldsymbol{\theta}} \pi(a|s,\boldsymbol{\theta})$$

$\mathbb{E}$ distribution of state induced by π

*Note how this theorem is non-trivial... The first two terms depends on π, but we did not take the gradient w.r.t. them.

Stochastic approximation in policy gradients

$$\nabla J(\boldsymbol{\theta}) = \sum_s d^\pi(s) \sum_a Q^\pi(s,a) \nabla_{\boldsymbol{\theta}} \pi(a|s,\boldsymbol{\theta})$$

- Sample from running policy π
  - $\underline{(S_1, A_1, R_1), \dots, (S_T, A_T, R_T)}$

Stochastic approximation in policy gradients

$$\nabla J(\boldsymbol{\theta}) = \overset{\mathbb{E}_{d^\pi(s)}}{\sum_s} d^\pi(s) \overset{\mathbb{E}_{a\sim\pi(\cdot|s)}}{\sum_a} Q^\pi(s,a)\nabla_{\boldsymbol{\theta}}\pi(a|s,\boldsymbol{\theta})$$

$$= \pi \cdot \nabla \log \pi$$

$$\mathbb{E}_{a\sim\pi(\cdot|s)}$$

- Sample from running policy π
  - $(S_1, A_1, R_1), \ldots, (S_T, A_T, R_T)$
- Idea: Sample s, then the following is an unbiased estimator (finite horizon episodic case)

$$\sum_{t=1}^{T} \Big(\underbrace{\sum_{\ell=t}^{T} R_\ell}_{\text{MC approximation}} \Big) \frac{\nabla_\theta \pi(A_t|S_t,\theta)}{\pi(A_t|S_t,\theta)}$$

$$\boxed{\nabla_\theta \log \pi = \frac{1}{\pi}\nabla \pi}$$

$$\nabla \pi$$

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$$\underline{= \sum_{t=1}^{T} G_t \nabla_\theta \log(\pi(A_t|S_t,\theta))}$$

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$$= \sum_{t=1}^{T} \underline{G_t} \nabla_\theta \log(\pi(A_t|S_t,\theta))$$

← REINFORCE

↑ Replace with TD approx. / Bootstrapping

***Show that this is an unbiased estimator of the gradient.**

Checkpoint for RL

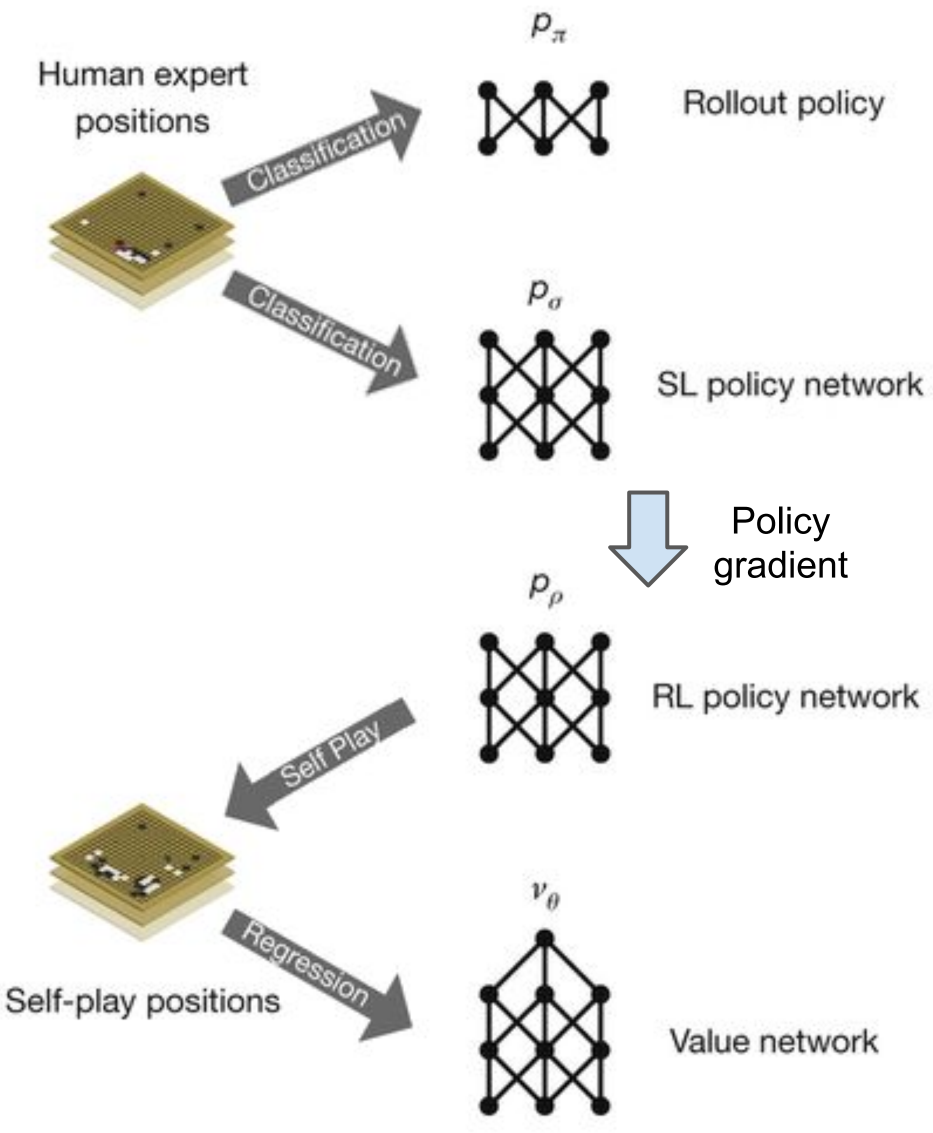
- Model-based methods
- Model-free methods
    - Monte Carlo methods
    - TD-learning: Q-Learning and Sarsa
- Function approximation in RL
    - Approximate the MDP: Model-based
    - Approximate the value function
- Policy gradients
    - Parametrize the policy and run SGD

Elements of State-of-the-Art Reinforcement Learning

- Use a deep neural network to parameterize Q-function
- Use a deep neural network to parameterize the policy \pi
- Run a combination of Q-learning and Policy Gradient.
  - Actor-Critics, A3C, etc…
- Heuristic-based exploration: curiosity, reward shaping, etc..
- Experience replay to generate more data from existing data.
- Multi-agent RL: modeling your opponents

Alpha-Go and Alpha-Zero

- Parameterize the policy networks with CNN
- Supervised learning initialization
- RL using Policy gradient
- Fit Value Network (This is a value function approximation)
- Monte-Carlo Tree Search



https://www.youtube.com/watch?v=4D5yGiYe8p4

D. Silver. Mastering the game of Go with Deep Neural Networks and Tree Search. Nature, vol. 529 issue 7587

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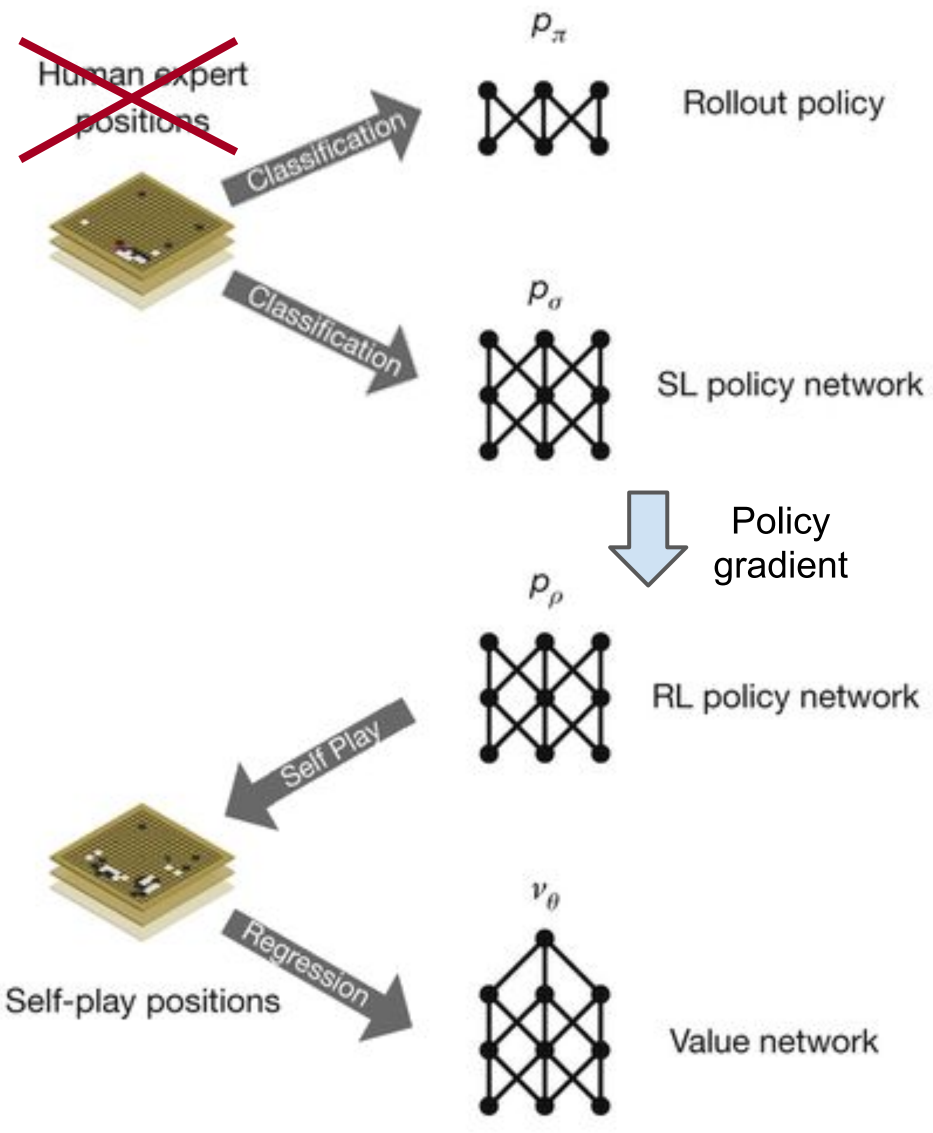

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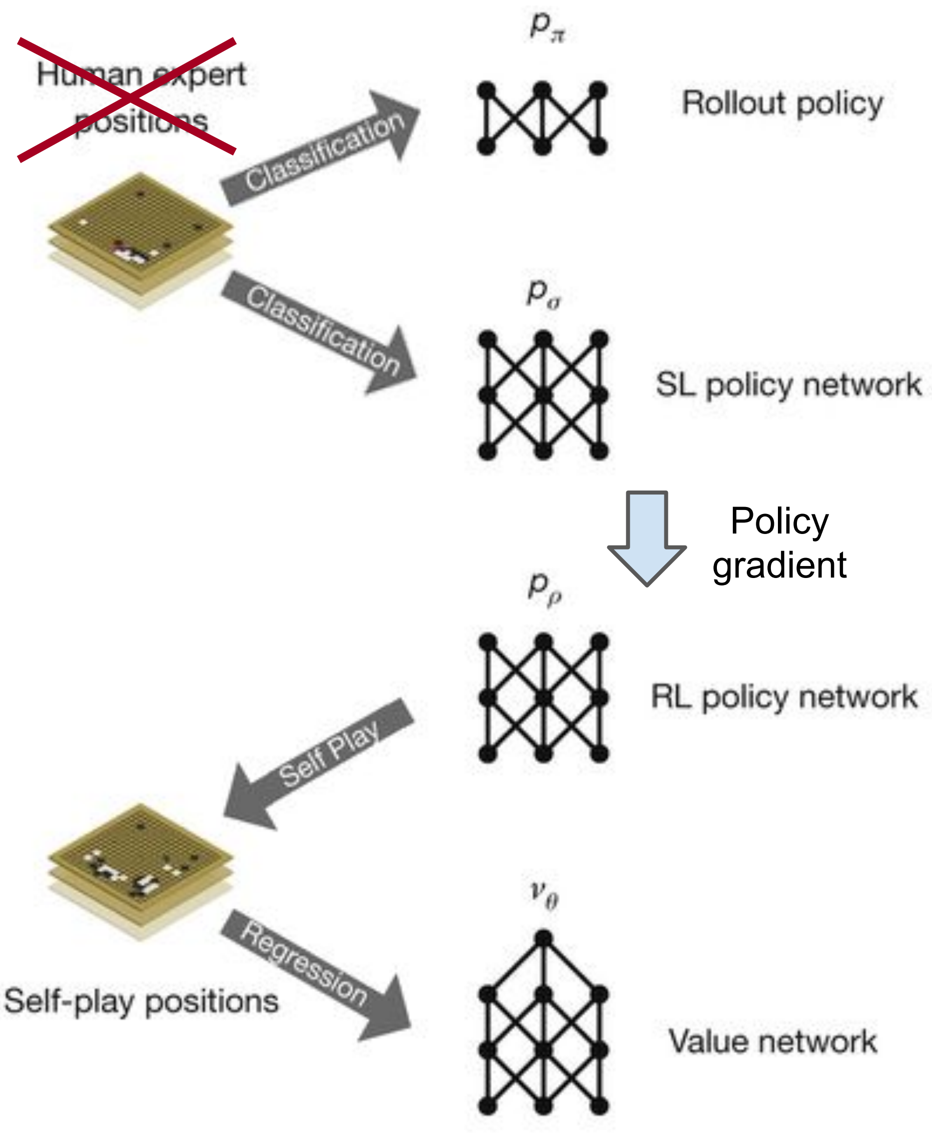

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D. Silver, et al. "Mastering the game of go without human knowledge." *Nature* 550.7676 (2017): 354-359.

What I did not cover

- Useful results in RL for both theory and alg design
  - Simulation lemma
  - Advantage function and performance difference lemma
- Exploration
  - “Optimism in the face of uncertainty”
- Offline RL
  - “Pessimism in the face of uncertainty”
- How to start research in RL ?
  - Take my RL course (email me to ask for the videos)
  - Solve homework problems, implement RL algorithms from scratch.

Final words to students

- If you are doing theoretical research
  - It’s useful have an empirical mind set
  - implement your algorithm, try it on examples (even toy examples would work)
  - These help you to challenge your assumptions and define theoretical problems that are useful

- If you are doing empirical research
  - Don’t just chase SOTA in benchmarks
  - Think deeply about the problems you are working on
  - ML theory helps you to avoid pitfalls and design better algorithms.

Thank you! Looking forward to your project presentations!